Mean mean-wind

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Abstract

Cup anemometers with pulse signal output are used in many meteorological observatories. The WMO's *Guide to Meteorological Instruments and Methods of Observation* suggests, and the framework of IEC-61400-50 and -12 standards stipulates, that its signal shall be sampled at least once per second and that these samples shall be averaged when seeking the 10-minutes' mean-wind speed.

The rule remains ambiguous about: (a) how the wind speed signal shall be sampled, and (b) how these samples shall be averaged?

For sampling, the obvious alternatives are: (i) to either count the signal's pulses during constant periods of time, or alternatively (ii) to measure the time elapsed between a constant number of pulses, in other words during a constant wind run.

Both methods render correct samples of wind speed. However, method (ii) differs from method (i) insofar as the number of samples depends on the generally varying wind speed instead of the constant advance of time. Hence, the mean-wind speeds obtained from the different sets of samples must differ too. The elementary relation between such different sample distributions, namely the wind speed itself, allows to derive one type of sample

distribution from the other. And likewise for their means: of speed, of linear impulse flux, and of kinetic energy flux too.

Both methods, and the resulting means, are equally legitimate by the mere word of the standard framework. However, confusion of these mean-wind speeds has lead to blunders, and unnecessarily cast doubts on the truthfulness of cup anemometry.

The guideline, the standard framework, and the traceability of calibration and classification procedures related thereto, can be improved by disambiguating the term wind speed: (i) which wind speed sample is actually meant in the context, is it the wind speed sampled at constant time interval v_i, or is it the wind speed sampled at constant wind run u_i, (ii) is one to be preferred over the other, (iii) by which method is their mean most advantageously derived, and (iv) does it matter how and what point the instrument´s raw signal is converted to speed?

Here, we investigate the intricate relation between these two types of wind speed samples and their most obvious means.

Contents

Nomenclature

$[v_i]$ sample i of the number of anemometer signal pulses counted during constant counter gate periods (e.g. counts during one second), converted to physical units m/s, also called *time-wise sample;* $v_i = \mathit{\Delta} l_i/\Delta tsa$, *where* $\mathit{\Delta} l_i =[\Delta n_{i,\mathit{\Delta} tsa} l_0]$

$[\overline{v}]$ arithmetic average of the time-wise samples v_i; the over-line denotes arithmetic averaging

$[u_i]$ sample i of time δt_i elapsed between consecutive pulses or their multiple (e.g. time elapsing between each cup revolution), converted to physical units m/s, also called *space-wise sample;* $u_i = \Delta lsa/\Delta t_i$

$[\overline{u}]$ arithmetic average of the space-wise samples u_i

$[T]$ period of the averaging interval for which the mean-wind speed is sought, e.g. 10 minutes

$[n]$ number of space-wise samples u_i acquired during the averaging period T

$[m]$ number of time-wise samples v_i acquired during the averaging period T

$[\mathit{\Delta} l_i]$ wind run pertaining to each sample when sampling time-wise

$[\Delta lsa]$ sampling length, constant length of wind run from which each space-wise sample u_i is acquired

$[l_0]$ wind run making the cup anemometer turn out one pulse (or the number of pulses marking a complete revolution); slope of the linear transfer function derived by calibration

$[L]$ total wind run reported by the cup anemometer during the averaging period T

$[\Delta t_i]$ time elapsing between consecutive signal pulses i and (i-1) (or a multiple of pulses, most advantageously representing one revolution of the cup star); the raw sample during space-wise sampling

$[\triangle tsa]$ sampling period, period after which the counter gate is reset (period after which a new time-wise sample is acquired (e.g. one second))

$[\Delta n_{i,\mathit{\Delta} tsa}]$ number of pulses counted during a sampling period Δtsa; the raw sample during time-wise sampling

$[\sigma_T]$ standard deviation of the time-wise wind speed samples acquired during an averaging period T; the respective coefficient of variation $\sigma_T/\overline{v}$ is known as Ti, *turbulence intensity*

[σ_S] standard deviation of the space-wise wind speed samples acquired during the same averaging period T as for the samples above

[$f_T(v)$] density function of the time-wise wind speed samples

[$f_S(v)$] density function of the space-wise wind speed samples

[Ti] the coefficient of variation - or relative standard deviation - of the time-wise wind speed samples v_i, viz. $\sigma_T/\overline{v}$, also known as the *turbulence intensity*

[$\overline{X}$] the over-line denotes arithmetic averaging (here the tuple X)

[$L_r(u_i)$] Lehmer mean of order r of the space-wise samples, viz.$\sum_{i=1}^{i=n} u_i^r / \sum_{i=1}^{i=n} u_i^{r-1}$

[$L_r(v_i)$] Lehmer mean of order r of the time-wise samples. viz. $\sum_{i=1}^{i=m} v_i^r / \sum_{i=1}^{i=m} v_i^{r-1}$

[$S_T(r)$] sum of the r^{th} power of time-wise wind speed samples; the r-th surface moment of the sample distribution

[$S_S(r)$] sum of the r^{th} power of space-wise wind speed samples; the r-th surface moment of the sample distribution

[Ke] energy pattern factor

1 Introduction

Cup anemometers are affordable, robust, and they can run virtually off the charge of a few electrons and without maintenance for years. Over the past one-hundred-and-fifty years, they have been deployed in the hundreds of thousands, in virtually all climates. Bright people have sought to understand the working of this instrument. Their insights are in the public domain, accessible to all, and open to discussion and challenge. Their effort has made the cup anemometer the ultimate reference in wind measurement. This will likely remain so, alone on grounds of transparency.

The WMO guideline and the IEC 61400-50 and -12 standard framework, which govern today´s wind power anemometry, are ambiguous. What follows from them, for devising a mean-wind speed measurement chain, is that the wind´s speed shall be sampled without interruption, more often than once per second, and then averaged for periods of 600 seconds, or over 60 seconds respectively for small wind turbines. This rule, implemented with the obvious choice of hardware, leads to that some types of measurement chains report the arithmetic mean of the wind speed sampled during constant time intervals[1]. The particular mean-wind speed derived in this way corresponds to a mean mass flow in the ever-changing direction of the wind, namely the wind speed´s time average, which we here call $\overline{v}$. Other measurement chains, sampling the wind speed at constant intervals along the wind run, and working perfectly in accordance with the above rule, report another mean wind speed, namely the space-average of the wind speed, here called $\overline{u}$. We shall see that this mean-wind speed is related to the wind´s mean impulse flux. The two mean-wind speeds are both legitimate in the sense of the framework, but they generally differ in magnitude.

Today, those deploying anemometric measurement chains, and those taking decisions on basis of cup anemometry, can hardly know for certain what sort of mean is being reported. Testing a measurement chain relies on applying a constant, known frequency from a calibrated signal generator to the data logger´s digital channel and on verifying the corresponding reading. This test will not reveal whether the wind speed will be sampled equidistant in time or equidistant in space, namely along the wind run. Neither would the verification of the mean of wind speed samples derived from a constant-frequency test signal reveal the method of sampling and averaging. Their combinations yield the same means

[1] The rule´s lower frequency limit hints to that this might have been tacitly intended.

as long as the frequency of the test signal remains constant over the averaging period.

Hence this attempt to clarify what can lead to blunders when following the standard framework to the mere word.

The clarification relies on the elementary relation between the wind speed´s time-average and its space-average along the wind run co-ordinate: The wind run per time changes with speed, and hence there will be more or less wind speed samples when sampling equidistantly along the wind run, whereas there will always be just one when sampling equidistantly in time. The space-average is generally larger in magnitude than its time-average, the more so the larger the wind speed´s variance. The difference in magnitude is small enough to be put off as uncertainty in *laminar* wind tunnels. In the free atmosphere, where the wind speed´s variance is much higher, the difference is large enough to cause blunders. For instance when referencing a wind turbine´s power curve to one of these mean-wind speeds, and estimating its average annual power production from the other. Such estimates can be biassed by several percent.

The difference of these mean-wind speeds has been pointed out by Leif Kristensen [1] in the context of the cup anemometer´s over-speeding, and practitioners of wind power anemometry have been aware of them for decades. Some of them have preferred one over the other. Yet, the standard framework does not discern these means, nor seems there any treatise addressed to the general public that shows the nature of their relation.

We hope to fill this void by illustrating their difference starting with an elementary arithmetic example.

Mind, the aforementioned ambiguity can be avoided altogether when the instrument´s raw signal, namely pulse counts, or time periods respectively[2], are first averaged, and only thereafter the result is converted to physical units of speed. This sequence, however, comes at the high cost of the means of the wind speed´s higher orders.

The following is exclusively concerned with sampling and sample aggregation in measurement chains reporting wind speed, minus the cup anemometer itself. The cup anemometer´s instrument theory, important in the context of wind power anemometry, is left untouched, namely: (i) speed-velocity bias - v-bias in Kristensen´s notion and DP error in McCready´s; (ii) overspeeding - u-bias an

[2]Namely the nominator or denominator of ds/dt separately.

w-bias, (iii) cup star wobble, and (iv) the limit of knowledge when observing wind at just one point. Literature on these subjects is suggested in the references.

For an understanding of this investigation, the reader ought not to know more of the cup anemometer, than that its cup star revolves once per constant wind run, no matter the speed.

2 Time-wise and space-wise speed samples

The magnitude of the arithmetic average of speed samples from a cup anemometer with pulse output depends on whether the wind speed samples have been derived

- at constant intervals of time, counting the pulses per each second, here called v_i, or
- at constant intervals of wind run, namely from measuring the time that elapses between a constant number of pulses, here called u_i[3].

Both of these arithmetic averages are meaningful and both have the unit of the samples: meter per second. The first-mentioned mean-wind speed is the wind speed´s time-average $\overline{v}$, and the second is the wind speed´s space-average $\overline{u}$, along the curvilinear wind run coordinate. The second is generally larger than the first. Just when the wind speed does not vary, both are of the same magnitude.

By *convention of habit* it is the time-average of the wind speed, here denoted by $\overline{v}$ that we usually mean when we speak of *mean speed*[4]; its equals the most evident of mean-wind speeds: the total wind run divided by the period of the averaging interval. We have less of a notion of the space-average of the wind speed, $\overline{u}$. We hope to redress this by showing (i) the physical quantity it is a measure of, and (ii) how it is related to time-average of the mean-wind speed.

Arithmetic example The relation between the wind speed´s time-average and its space-average may be illustrated by the following arithmetic scheme: Let´s assume the wind blows at discrete wind speeds, with sudden transitions between these speeds. The wind prevails at one speed for a certain time, then it suddenly blows at another speed for another period of time, after which it blows at another speed for another period, and so on. A cup anemometer is following the wind speed without delay, emitting a pulse at each full revolution, which corresponds to a wind run of, say, 1 metre. The wind samples are computed from the anemometer´s pulse output in two ways: (i) time-wise, by counting the pulses for each second, Δtsa = 1 s, and converting them to physical units of speed, and (ii)

[3]We do not consider the mixed case here, where both the time and wind run interval are varying, where time-wise and space-wise samples are aggregated to one mean, or where the set of samples is re-sampled in time or in space.

[4]The over-line denotes arithmetic averaging throughout this note.

space-wise, by measuring the time between consecutive pulses, hence Δlsa = 1 m, and converting them to physical units.

Let´s assume, this wind possesses a step-wise anemograph $v(t)$ rectangular over time: 0.5 m/s for 2 seconds, 1 m/s for 2 seconds, 2 m/s for 1 second, 3 m/s for 3 seconds, and 4 m/s for 1 second:

speed step number	1	2	3	4	5	row´s sum
speed [m/s]	1/2	1	2	3	4	-
wind run that each step contributes [m]	1	2	2	9	4	18
time period that each step prevails [s]	2	2	1	3	1	9

Table 1: Exemplary step-wise anemograph

Time-wise sampling results in the following samples and a time-average of the mean-wind speed of 2.00 m/s:

speed step number	1	2	3	4	5	row´s sum
time-wise samples v_i [m/s]	1/2, 1/2	1, 1	2	3, 3, 3	4	18
number of time-wise samples m [-]	2	2	1	3	1	9
wind speed´s time-average $\overline{v}$: arithmetic average of time-wise samples = 18/9 = 2.00 m/s						

Table 2: Time-wise sampling and resulting mean-wind speed: 2.00 m/s

Space-wise sampling results in the following samples and a space-average of the mean-wind speed of 2.75 m/s:

step number	1	2	3	4	5	row´s sum
space-wise samples u_i [m/s]	1/2	1, 1	2, 2	3, 3, 3, 3, 3, 3, 3, 3, 3	4, 4, 4, 4	49+1/2
number of space-wise samples n [-]	1	2	2	9	4	18
wind speed´s space-average $\overline{u}$: arithmetic average of space-wise samples = 49,5/18 = 2.75 m/s						

Table 3: Space-wise sampling and resulting mean-wind speed: 2.75 m/s

The relation between the sample distributions is illustrated in table 4 below. The speed is binned to 0.5 m/s. The first bin contains samples for wind speeds between 0.25 and 0.75 m/s, the second contains the samples from 0.75 to 1.25 m/s, and so on. For one and the same anemograph given in table 1, the number of time-wise samples from table 2 and the number of space-wise samples n_{bin} from table 3 are counted.

0	bin centre speed v_{bin} [m/s]	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	row´s sum
1	number, time-wise samples m_{bin}	2	2	0	1	0	3	0	1	9
2	number, space-wise samples n_{bin}	1	2	0	2	0	9	0	4	18
3	$n_{bin} = m_{bin} v_{bin}$	1	2	0	2	0	9	0	4	18

Table 4: Example: Bin centre speed, bin-wise number of time-wise and space-wise samples and their relation (row 3)

Rows 1 and 2 represent the histograms of the time-wise samples, and space-wise samples respectively. Row 3 shows the bin-wise product of the number of speed-wise samples times the bin speed $n_{bin} = m_{bin} v_{bin}$. Note that for each bin row 3 equals the number of space-wise samples.

For simplicity, the sampling period Δtsa as well the sampling length Δlsa have both been assumed unity. For arbitrary sampling periods and sampling lengths, the relation between sample counts turns out:

$$n_{bin} = \frac{\Delta tsa}{\Delta lsa} \cdot v_{bin} \cdot m_{bin}. \tag{1}$$

Equation 1 allows to build the histogram of time-wise samples from the histogram of space-wise samples and vice versa.

Generalised relation for continuous variable When replacing discrete samples and histograms in the arithmetic example above by continuous variables and their density functions, $f_T(v)$ for time-wise samples and $f_S(v)$ for space wise samples respectively, equation 1 turns into $f_S(v) = cvf_T(v)$.[5]

[5] In anemometric practise, speed samples cannot be continuous variables in the sense of Pierre Varignon´s definition of instantaneous speed as a ratio of continuous infinitesimals, $v = ds/dt$. Actually, the sampling period and the sampling length are always multiples of a constant. Hence the speed samples are discrete, rational numbers, however small their increment. (In practical cup anemometry practise, time-wise sampling will lead to a more coarse quantisation than space-wise sampling, since the time increment is of the order of just microseconds whereas the space increment is of the order of metres. Space-wise sampling limits quantisation noise, which would mimic variance.)

Let´s use Varignon´s definition of *instantaneous* speed: $v = ds/dt$, and namely $dl = vdt$.

For instance: If dt were a constant 1 second, the wind run dl increases by 10 when the speed changes from 1 to 10 metres per second. When taking each sample at a constant wind run, their number will be v times larger than the sample that is taken at each second. Hence

$$f_S(v) = cvf_T(v) \tag{2}$$

where c denotes an unknown coefficient.

From the frequency distribution $f_T(v)$ of the time-wise samples follows the wind speed´s time-average

$$\overline{v} = \int_0^\infty vf_T(v)dv, \tag{3}$$

and from the frequency distribution $f_S(v)$ of the space-wise samples follows the wind speed´s space-average

$$\overline{u} = \int_0^\infty vf_S(v)dv. \tag{4}$$

Substituting $f_S(v)$ according to equation 2 reveals

$$\overline{u} = \int_0^\infty cv^2 f_T(v)dv, \tag{5}$$

that the wind speed´s space average is a function of the wind speed‘s second order.

To find the relation between the time-average and the space-average we must find the constant c: By definition, the integral of a probability density function is one and so.

$$\int_0^\infty f_S(v)dv = 1 \tag{6}$$

and since

$$\int_0^\infty f_S(v)dv = c\int_0^\infty vf_T(v)dv \tag{7}$$

it follows after insertion of equations 3 and 6 into 7 that $c\overline{v} = 1$, and $c = \frac{1}{\overline{v}}$.

Substituting c in equation 5 by $1/\overline{v}$ shows

$$\overline{u} = \frac{1}{\overline{v}} \int_0^\infty v^2 f_T(v) dv = \frac{\overline{v^2}}{\overline{v}} \tag{8}$$

that the space-average of the wind speed is the proportion of the time-average of the wind speed squared divided by the time average of its speed.

The above holds for any order r of the speed v, and reveals the general relation between the time averages and the space averages:

$$\overline{u^r} = \frac{1}{\overline{v}} \overline{v^{r+1}}. \tag{9}$$

The relation hints at that the mean impulse and the mean kinetic energy flux can be construed from means alone.

Space-wise and time-wise sample distributions relate to one another alike
a1)

$$f_S(v) = \frac{1}{\overline{v}} v f_T(v),$$

which for sets of finite samples corresponds to
a2)

$$n_{bin} = \frac{\mathit{\Delta} tsa}{\mathit{\Delta} lsa} \cdot v_{bin} \cdot m_{bin}$$

and their respective means of whatever order are related by
b)

$$\overline{u^r} = \frac{1}{\overline{v}} \overline{v^{r+1}}.$$

By the help these relations, the samples and means can be converted from one to another, and represented in various ways:

	time-average mean-wind speed $\overline{v}$	space-average mean-wind speed $\overline{u}$	expressed in terms of
1	$\frac{n\Delta lsa}{T}$		sum of wind run divided by time
2	$\frac{1}{m}\sum_{i=1}^{m} v_i = \frac{1}{m}\sum_{1=1}^{m} \frac{\Delta l_i}{\Delta t_{sa}}$	$\frac{1}{n}\sum_{i=1}^{n} u_i = \frac{1}{n}\sum_{1=1}^{n} \frac{\Delta lsa}{\Delta t_i}$	samples´ raw moments = arithmetic averages
3	$\sum_{i=1}^{n} u_i \frac{\Delta t_i}{T}$	$\overline{v}\left[1+\left(\frac{\sigma_T}{\overline{v}}\right)^2\right]=\frac{\overline{v^2}}{\overline{v}}$ $=\overline{v}\left[1+Ti^2\right]$	turbulence intensity (for $\overline{u}$ only)
4	$\left[\frac{1}{n}\sum_{i=1}^{i=n}\frac{1}{u_i}\right]^{-1}$	$\frac{\sum_{i=1}^{m} v_i^2}{\sum_{i=1}^{m} v_i}$	harmonic and contra-harmonic means
5	$\int_0^\infty v f_T(v)dv$	$\frac{1}{\overline{v}}\int_0^\infty v^2 f_T(v)dv$	density functions
6	$L_1(v_i) = L_0(u_i)$	$L_1(u_i) = L_2(v_i)$	Lehmer means (see below)
7	$A_T\Gamma\left[1+1/k_T\right]$	$A_T\frac{\Gamma[1+2/k_T]}{\Gamma[1+1/k_T]}$	the Gamma function and parameters of the 2-parametric Weibull distribution A_T and k_T

Table 5: Relations between time-average and space-average of the wind speed

The expression for $\overline{v}$ in row 4 was pointed out by Kristensen [1, 2], as well as the expression for $\overline{u}$ in row 3 [6].

Likewise, the mean-wind speeds of higher order, the wind´s linear impulse´s time-average $\overline{I_T}$ and the time-average of its kinetic energy flux $\overline{E_T}$ can be derived from either time-wise or from space-wise samples:

[6]Exploiting the definition of the variance, here for the time-wise samples, namely $var(v) = \overline{v^2} - \overline{v}^2$ we may express the proportion as

$$\frac{\overline{v^2}}{\overline{v}} = \overline{v}(1 + \frac{var(v)}{\overline{v}^2}) = \overline{u}.$$

The square root of their variance is the standard deviation $\sigma = \sqrt{var(v)}$, and the coefficient of variation $\sigma/\overline{v}$ is known as the turbulence intensity Ti. Hence, the proportion between the space-average and the time-average is $1 + Ti^2$, the proportion found by Kristensen, in this context, and also as a measure for the cup anemometers´ u-bias.

	time-average mean-wind speed $\overline{v}$	space-average mean-wind speed $\overline{u}$	linear impulse flux $\approx \overline{I_T}$	kinetic energy flux$\approx \overline{E_T}$
1	$\frac{nl_{sa}}{T}$	$\frac{\overline{v^2}}{\overline{v}}$	$\overline{v^2}$	$\overline{v^3}$
2		$\overline{u}$	$\overline{v}\overline{u} = \frac{n\Delta l_{sa}}{T}\overline{u}$	$\overline{v}\overline{u^2} = \frac{n\Delta l_{sa}}{T}\overline{u^2}$
3	$\frac{1}{m}\sum\limits_{i=1}^{m} v_i$	$\frac{1}{n}\sum\limits_{i=1}^{n} u_i$	$\frac{\Delta l_{sa}}{T}\sum\limits_{i=1}^{n} u_i = \frac{1}{m}\sum\limits_{i=1}^{m} v_i^2$	$\frac{\Delta l_{sa}}{T}\sum\limits_{i=1}^{n} u_i^2 = \frac{1}{m}\sum\limits_{i=1}^{m} v_i^3$

Table 6: Time-averages and space-averages of speed ; and time-averages of speed, linear impulse and kinetic energy flux

The fact that the time-average of the linear impulse flux is the wind speed´s space-average times its time-average has been pointed out by Terry McConnel [3]. The mean impulse flux can be interpreted as the mean mass flow times the mass-weighted mean-wind speed.

Further generalisation

For abbreviation, let´s call the sums of the samples[7] to the order of r $\sum\limits_{i=1}^{m} v_i^r = S_T(r)$ and $\sum\limits_{i=1}^{n} u_i^r = S_S(r)$.

The Lehmer mean $L_r(X)$ of the r-th order of the tuple X of positive real numbers is defined as

$$L_r(X) = \frac{\sum_i X_i^r}{\sum_i X_i^{r-1}} = \frac{S_X(r)}{S_X(r-1)} \tag{10}$$

namely as the ratio of the sums of the samples of falling order[8]. The Lehmer mean of the r-th order of the time-wise samples is

$$L_r(v_i) = \frac{S_S(r)}{S_S(r-1)} \tag{11}$$

and that of the space-wise samples is

[7] The sums of the order r correspond to the r-th surface moment of the samples.

[8] The ancient methods of averaging can be expressed by Lehmer means, especially:
$L_0(X)$ is the harmonic mean; and
$L_1(X)$ is the arithmetic mean, both Babylonian means; and
$L_2(X)$ is the contra-harmonic mean, the mean of Eudochos of Kinidos; and
$L_{1/2}(X)$ is the geometric mean, a Pythagorean mean.

$$L_r(u_i) = \frac{S_T(r)}{S_T(r-1)}. \tag{12}$$

We further know that the sums of the time-wise samples to the order $r+1$ are proportional to the sums of the space-wise samples to the order of r. When the sums of time-wise and space-wise samples of adjacent order are related, their proportions must be related too.

Indeed, the r-th Lehmer mean $L_r(u_i)$ of space-wise samples of the order r equals the $r+1$st Lehmer mean of time-wise samples of the order $r+1$, viz. $L_r(u_i) = L_{r+1}(v_i)$

since generally

$$\frac{\sum u_i^r}{\sum u_i^{r-1}} = \frac{S_S(r)}{S_S(r-1)} = \frac{\sum v_i^{r+1}}{\sum v_i^r} = \frac{S_T(r+1)}{S_T(r)}. \tag{13}$$

This implies for the two mean-wind speeds: *The harmonic mean of the space-wise samples, $L_0(u_i)$, equals the arithmetic mean of the time-wise samples, $L_1(v_i)$, and both represent the wind speed´s time-average $\overline{v}$.*

The space-averages and time-averages of the higher orders of speed samples, notably the time-average of the impulse flux and the time-average of the flux of kinetic energy too, can be estimated from Lehmer means. The product series of r consecutive Lehmer means

$$\prod_{p=1}^{p=r} L_p(v_i) = \frac{\sum v_i^1}{\sum v_i^0} \cdot \frac{\sum v_i^2}{\sum v_i^1} \cdot \frac{\sum v_i^3}{\sum v_i^2} ... \cdot \frac{\sum v_i^r}{\sum v_i^{r-1}} = \frac{\sum v_i^r}{\sum v_i^0} \tag{14}$$

equals the time-average of the r-th moment of the wind speed samples. (Cross out equal terms in nominator and denominator.)

The same magnitudes[9] can be derived from the space-wise samples, by reducing the order r by 1, since $L_r(v_i) = L_{r-1}(u_i)$:

$$\prod_{p=0}^{p=r-1} L_p(u_i) = \frac{\sum u_i^0}{\sum u_i^{-1}} \cdot \frac{\sum u_i^1}{\sum u_i^0} \cdot \frac{\sum u_i^2}{\sum u_i^1} ... \cdot \frac{\sum u_i^{r-1}}{\sum u_i^{r-2}} = \frac{\sum u_i^{r-1}}{\sum u_i^{-1}} = \frac{\sum v_i^r}{\sum v_i^0}. \tag{15}$$

Each of these three mean quantities, for mass flow, impulse flux and kinetic energy flux, corresponds to one particular mean wind speed, derived from time-wise

[9] It might interest some that the mean kinetic energy flux equals the product of the harmonic mean, the arithmetic mean and the contra-harmonic mean of the space-wise wind speed samples.

samples

$$\left[\prod_{p=1}^{p=r} L_p(v_i)\right]^{1/r} = \left[\frac{\sum v_i^r}{\sum v_i^0}\right]^{1/r}, \tag{16}$$

or from space-wise samples

$$\left[\prod_{p=0}^{p=r-1} L_p(u_i)\right]^{1/r} = \left[\frac{\sum u_i^{r-1}}{\sum u_i^{-1}}\right]^{1/r}, \tag{17}$$

for $p = 1$: the well-known mean-wind speed in time, $\overline{v}$, representing the mass flow, and proportion of total wind run and averaging period L/T;

for $p = 2$: the mean-wind speed in space, let´s call it $\overline{i}$, that, when squared, corresponds to the mean flux of linear impulse $\overline{uv}$;

for $p = 3$: another mean-wind speed, let´s call it $\overline{e}$ that, when cubed, corresponds to the mean kinetic energy flux;

and so on.

Each of these three pairs of physically meaningful mean-wind speeds represents the average of a flow quantity. Any of them may be calculated purely from time-wise samples, as per equation 16, the other from space-wise samples, as per equation 17, or from a combination both types of samples. All three measure a mean quantity that potentially is conservative: mass, linear momentum and energy[10].

[10]The latter two, which may be composed of both types of samples, are represented by geometric means.

3 Measuring...

Order of conversion and averaging The standard framework remains largely ambiguous about the nature of the samples that should be averaged. The standard bids averaging wind speeds, hinting to that the raw observation shall be converted to physical units of speed before averaging. Mind that the raw observations are, either (i) pulse counts $\Delta n_{i.\Delta tsa}$ during the sampling period Δt_{sa}, or (ii) time periods Δt_i between two pulse flanks. If such raw observations were averaged first, and their average converted to speed thereafter, there would be just one and the same mean-wind speed, irrespective of whether the raw observation has been sampled time-wise or space-wise. Averaging either of the raw observations yields the time-average of the mean-wind speed $\overline{v}$, the elementary, *naïve* mean-wind speed S/T.

The $\overline{u} - \overline{v}$-ambiguity may be avoided, albeit at the high cost of losing insight into the higher orders.

Pairing of sampling and averaging The standard framework ought to state that the way of averaging speed samples depends on the way the samples have been derived, either in time or in space. Averaging arithmetically by mere habit may inadvertently lead to space-averaged mean-wind speeds that are, at least for the moment, tacitly precluded when seeking a wind turbine´s cumulated energy output.

Sampling in time or in space? The time-wise and space-wise sample distributions and their means may be converted, using:

a1) the relation between the respective density functions or histograms

$$f_S(v) = \frac{1}{\overline{v}} v f_T(v),$$

which, for sets of finite samples, corresponds to
a2) the relation between the histograms of the discrete samples

$$n_{bin} = \frac{\Delta tsa}{\Delta lsa} \cdot v_{bin} \cdot m_{bin}$$

b) and the relation between the respective mean-wind speeds of whatever order

$$\overline{u^r} = \frac{1}{\overline{v}} \overline{v^{r+1}}.$$

The time-averages of the flow quantities most relevant in wind power engineering may be derived by just logging the time periods Δt_i and by deriving thereof, for each averaging interval, the pulse count n, and the space-average $\overline{u}$ of the wind speed.

mean-wind speed $\overline{v}$	mean impulse flux $\overline{I_T}$	mean kinetic energy flux $\overline{E_T}$	turbulence intensity $Ti = \sigma/\overline{v}$
$\frac{n\Delta lsa}{T}$	$\overline{v} \cdot \overline{u}$	$\approx \overline{u}^3$	$\sqrt{\frac{\overline{u}-\overline{v}}{\overline{v}}}$

Table 7: Time-averages of speed, linear impulse, kinetic energy flux and time-wise turbulence intensity derived from the number and n of space-wise samples and the space-average wind speed $\overline{u}$

The mean kinetic energy flux may further be approximated[11] as $\overline{E_T} \approx \overline{u}^3$.

Mean-wind speeds and their ranking

The mean-wind speeds characterising the flow and their distance from $\overline{v}$ are presented here:

[11] $\overline{v^3} = \frac{1}{n}\sum(\overline{v}+\Delta v_i)^3 = \frac{1}{n}\sum(\overline{v}^3 + 3\overline{v}(\Delta v_i)^2 + (\Delta v_i)^3) \approx \overline{v}^3(1+3\frac{\sigma^2}{\overline{v}^2})$
$\overline{u}^3 = \overline{v}^3(1+\frac{\sigma^2}{\overline{v}^2})^3 = \overline{v}^3(1+3\frac{\sigma^2}{\overline{v}^2}+3\frac{\sigma^4}{\overline{v}^4}+\frac{\sigma^6}{\overline{v}^6})$ $\overline{v^3} \approx \overline{v}^3(1+3\frac{\sigma^2}{\overline{v}^2})$ and hence $\overline{v^3} \approx \overline{u}^3$
The wind speed´s space average cubed is a more truthful estimate for the mean kinetic energy flux than the wind speed´s time-average cubed.

mean speed designation	$\overline{v}$	$\overline{i}$	$\overline{u}$	$\overline{e}$
from time-wise samples v_i	$L_1(v_i)$ arithmetic mean	$\sqrt{\overline{I_T}} = \sqrt{\overline{v} \cdot \overline{u}} = \sqrt{\overline{v^2}}$ $\left[\prod_{p=1}^{p=2} L_p(v_i)\right]^{1/2}$ quadratic mean	$L_2(v_i)$ contra-harmonic mean	$\overline{v^3}$ $\left[\prod_{p=1}^{p=3} L_p(v_i)\right]^{1/3}$
from space-wise samples u_i	$L_0(u_i)$ harmonic mean	$\left[\prod_{p=0}^{p=1} L_p(v_i)\right]^{1/2}$	$L_1(u_i)$ arithmetic mean	$\left[\prod_{p=0}^{p=2} L_p(v_i)\right]^{1/3}$
physical meaning (examples)	speed´s time-average	square root of impulse´s time average	speed´s space average, impulse´s time average divided by speed´s time average	impulse´s space average divided by speed´s space average, third root of energy´s time average $\overline{E_T}$
approximate distance from $\overline{v}$	0	$\overline{v}(\left[\sqrt{1+Ti^2}-1\right]$	$\overline{v}Ti^2$	$Ke^{1/3}$

Table 8: Mean-wind speeds ranked by their distance from the time-averaged mean-wind speed S/T

Turbulence intensity The coefficient of variation of the time-wise wind speed-samples $\sigma_T/\overline{v}$, also known as the turbulence intensity Ti, can be derived from these means, directly as

$$Ti = \sqrt{\frac{\overline{u}}{\overline{v}} - 1} = \sqrt{\frac{L_2(v_i)}{L_1(v_i)} - 1} = \sqrt{\frac{L_1(u_i)}{L_0(u_i)} - 1}.$$

4 Conclusion

An important standard framework for wind power engineering, IEC 61400-12-x and -50-x, remains vague with regard to its primordial measurand, the mean-wind speed. Its has been shown, at the example of the cup anemometer with pulse output, that, when following the standard to the word, at least two different mean-wind speeds can be derived by arithmetic averaging of the wind speed samples, depending on whether they are derived by sampling the pulse signal equidistantly in time or equidistantly in space, along the wind run: the first type of samples yields the wind speed´s time-average; the latter its space-average. The space-average is generally larger than the time-average, and by how much is a function of the wind speed´s variance. For wind in the free atmosphere, and likely for calibration in *laminar* wind tunnels too, the distance between these means is large enough to cause blunders. Confusing one mean-wind speed with the other will cast unnecessary doubt on the truthfulness of cup anemometry. We therefore advocate to disambiguate at least these two mean-wind speeds.

The simple relation between density distribution of time-wise and space-wise samples, $f_S(v) = \frac{1}{\bar{v}} v f_T(v)$, allows to reconcile the mean-wind speeds reported by measurement chains that employ different methods of sampling, sample conversion and averaging. The means of the wind speed´s higher orders, namely those representing impulse flux and kinetic energy flux, are revealed as composed of means in space and means in time.

When seeking to measure the mean flow quantity most relevant to a particular fluid dynamic quest, sampling in space appears the more advantageous than sampling in time. This is owed (i) to that each sample corresponds to a parcel of roughly equal mass, (ii) to the reduction of quantisation noise (iii), and to the ease of jointly sampling a cup anemometer and other instruments.

Outlook In the context of wind energy conversion it is noteworthy that the time-average of the kinetic energy flux is more accurately approximated by the cube of the wind speed´s space-average than by the cube of its time average. This is not surprising, for a mean cubed is generally smaller than the mean of the third order of its samples. Rather, what surprises is: that the wind speed´s time average still appears more closely correlated with a wind energy converter´s mean output.

We speculate a mean-wind speed more truthful can be found. For instance by clarifying: how is *space*, along the wind run, expressed in the wind converter´s coordinates? This emphasises the importance of Kristensen´s notions of u-bias and v-bias. They, and the means discussed here, may pave the way for yet higher precision in wind power anemometry and in power production estimates.

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